# TECHNICAL NOTES

## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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# AN INVESTIGATION OF AIRPLANE LANDING SPEEDS

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S u m m a r y

This paper describes an investigation on airplane landing speeds which was made to determine the applicability of accepted aerodynamic theory to the prediction of this particular performance characteristic.

The experimental work consisted in measuring the landing speed of several monoplanes by a new photographic method. The results of these tests supplemented by available information regarding biplanes were compared with predictions made in accordance with basic aerodynamic theory. The prediction makes use of the fundamental relation between wing loading, lift coefficient, and speed of level flight, and the effects of aspect ratio and proximity to the ground on lift curve slope.

Very satisfactory agreement was found between the predicted and measured values.

A set of generalized lift curves developed to simplify the method of prediction are presented in an appendix.

# Introduction

Probably less is known and more is erroneously stated about the landing speed than is the case for any other airplane performance characteristic. · The prevalence of obviously erroneous and sometimes even absurd specifications has been pointed out in References 1 and 2. The state of knowledge is well indicated by the somewhat common practice of predicting landing speeds on the basis of maximum lift coefficients determined by model airfoil tests made at low Reynolds Number, no account being taken of the angle of attack in the landing attitude. It appears that this situation may be largely attributed to the lack of a simple method for obtaining experimental information on the subject.

The only known source of accurate information on airplane landing speeds is given in Reference 4. Tests of a number of military biplanes are described in this report but no analysis of the results is included.

The program of the present investigation was to supplement the available data with the results of tests on various types of monoplanes, to make predictions of landing speeds for all the tested airplanes and to compare the predicted and measured values.

## Experimental Methods and Apparatus

The biplane landing speeds reported by Carroll were determined by the use of synchronized recording flight instruments, namely, an air-speed meter, an accelerometer to indicate the instant of impact, and a ground-speed meter used as an air-speed check and to furnish wind-speed data. It is not definitely stated in the report whether a single representative landing or an average of several tests furnished the final results. The degree of accuracy is stated as plus or minus 3 per cent.

The method used by the writer was to determine the ground speed from a photographic record of the rotation of the landing wheel, by the use of a motion-picture camera which operated at a known constant speed, to measure the wind speed and to fix the still air landing speed as the algebraic sum of these quantities.

The procedure was to paint the wheel or tire of the airplane in contrasting colors, thus providing a rotation indicator, and then to photograph the airplane while making normal landings; wind speeds were simultaneously read from an anemometer. The duration of filming was from a second or two before to four or five seconds after impact.

The film was examined frame by frame, the position of the wheel in each picture recorded and a curve of wheel revolutions vs. exposures was plotted from these data. Knowledge of the

camera speed and tire circumference makes this curve in effect, a space-time curve from which velocities may be derived by graphical differentiation. It was found that the wheel acquired within one revolution a speed which varied negligibly during the next second or two. The slope of this portion of the curve was therefore taken to represent the speed at contact.

The motion-picture camera used was a portable, spring-driven type which operated at approximately 34 frames per second. A special electric clock was used to calibrate the camera. The exposure rate was determined for all conditions under which tests were likely to be made, i.e., from end to end of a full loading of film, and check calibrations were made at the beginning and end of each spool of film or each set of tests.

A vane, or windmill type of anemometer was used to measure the ground wind speed. Since this type of instrument registers only the revolutions made by the vanes, the wind speeds actually are average velocities over a period of about 10 seconds which includes the instant of landing.

## Method of Prediction

The method of prediction is based on the following conception of a perfectly executed three-point landing. The airplane approaches the landing area in a steady glide. On nearing the ground the flight path becomes practically horizontal, the angle of attack increasing and the speed decreasing in such a way as

to maintain constant lift until the attitude for the three-point landing (wheels and tail skid at the same height) is reached. This attitude is maintained while the speed and lift decrease sufficiently to let the airplane settle to the ground. The landing speed is therefore assumed to be the speed of level flight at a very small height above the ground at the angle of attack corresponding to a three-point landing.

It is known that upon approaching the ground there is a reduction of the induced angle of attack of the wings of an airplane which may be calculated according to the multiplane theory of Prandtl. This produces an increase of lift curve slope and a reduction of induced drag; it is therefore equivalent to an increase in the aspect ratio.

The calculation of this effective aspect ratio which is representative of level flight just above the ground is the first step of the prediction. It is next necessary to develop a curve of lift coefficient vs. angle of attack for a wing of this aspect ratio. The lift coefficient which the wings may be expected to attain is the value indicated by the curve at the angle of attack corresponding to a three-point landing.

It is now necessary to make some assumption regarding the relation of wing lift to the total lift of the airplane for the landing condition. It does not appear reasonable to assume that the two are equal because the sum of the vertical air loads on the fuselage and horizontal tail surfaces is usually too

large to be neglected. Since the air forces on the tail sur-
faces at high angles of attack and with conventional center of
gravity location are small, and since the lift of a normal type
of fuselage is considerable at high angles of attack, it is
reasonable to assume that the wings support somewhat less than
the total weight of the airplane. The lift of the parasite
parts might be approximated by crediting the airplane with the
wing area it would have if the wing were not interrupted, by
the fuselage. For the prediction of landing speeds, then, it
is assumed that the effective wing area is that of the complete
plan form of the wings, i.e., no deduction will be made for area
blanketed by the fuselage.

The required data, namely, weight, wing area, and lift
coefficient now being known, the landing speed is predicted
according to the equation

$$V = \sqrt{\frac{2\,W}{\rho\,C_L\,S}}$$

wherein
$V$ = velocity (feet per second)

$W$ = weight of airplane (pounds)

$\rho$ = density of air (slugs per cubic feet)

$C_L$ = lift coefficient

$S$ = wing area (square feet).

The effective aspect ratio corresponding to level flight
just above the ground is calculated as follows. The reduction

of the induced angle of attack of a wing system upon approaching

the ground is

$$\Delta \, \alpha_i = -\overline{\sigma} \, \frac{C_L \, S}{\pi \, {b_1}^2}$$

wherein $\qquad\qquad\Delta \, \alpha_i =$ change of induced angle of. attack (radians)

$\qquad\qquad\qquad\qquad\quad b_1 =$ maximum span (feet)

$\qquad\qquad\qquad\qquad\quad \overline{\sigma} =$ influence coefficient.

For the monoplane, $\overline{\sigma}$ is equal to $\sigma$, the coefficient of

mutually induced drag.* For the biplane $\overline{\sigma}$ is given by the

equation

$$\overline{\sigma} = \frac{\sigma_{1-1}{''} + \sigma_{2-2}{'} + 2 \, \sigma_{1-2}{''}}{4}$$

The subscripts follow the convention used by Wieselsberger

(Reference 3), that is, $\sigma_{1-1}{'}$ is the influence coefficient

for a biplane composed of one of the wings of the biplane and

its mirror image in the ground plane, $\sigma_{2-2}{'}$ corresponds to a

similar biplane composed of the other wing and its image, and

$\sigma_{1-2}{'}$ refers to a biplane composed of one of the real wings

and the mirror image of the other.

Strictly speaking, the above formulas are applicable only

to monoplanes and equal-span biplanes having ideal lift distri-

bution. However, the expressions may be used in practice with

no appreciable error for all normal biplanes and excepting on-

ly the extreme sesquiplane types.

From the equation for the induced angle of attack of a

*See Appendix II for values of $\sigma$.

monoplane

$$\alpha_i = \frac{C\ S}{\pi\ b^2}\ \left(or\ \frac{C_L}{\pi\ R}\right)$$

it may be deduced that the equivalent aspect ratio when near the ground is

$$R_e = \frac{R}{1 - \sigma}$$

and an analogous derivation for the biplane gives

$$R_e = \frac{R}{1 - \overline{\sigma}\ k^2}$$

R  =  aspect ratio of the real or equivalent monoplane.

$R_e$ =  effective aspect ratio of real or equivalent monoplane near the ground.

k  =  Munk's biplane constant.

The determination of the lift curve corresponding to this effective aspect ratio is accomplished by application of the well-known formula

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

to the results of full scale (Variable Density Wind Tunnel) tests of the airfoil profile used in the airplane.  In this formula

$\alpha_2$  =  angle of attack of airplane wing (degrees).

$\alpha_1$  =  angle of attack of airfoil model tested (degrees).

$R_1$  =  aspect ratio of model tested (after correction for tunnel wall effect).

$R_e$  =  effective aspect ratio of airplane wing (or that of equivalent monoplane).

## Results

The results of the investigation are presented in full in Appendix I; a condensed summary will be found in the next section.

The results of the experiments and calculations, together with the important design characteristics of the airplanes, are given in Tables II and III. The computations of the predicted landing speeds occupy pages 17-37, while the predicted lift curves are shown in Figures 1-11.

Examples of the film records appear as Figures 16-19.

Typical curves of wheel revolutions vs. exposures as derived from the pictures are presented as Figures 12-15.

The monoplane results were derived from only such landings as were found by examination of the films to be normal three-point landings. The average of at least two such landings was used as the final result for each airplane. The landing angles, wing areas, and weights were measured at the time of making the tests, but in some cases the angle of zero lift was obtained from the manufacturer.

## Discussion

For convenience of reference a summary of the results is presented in Table I.

TABLE I

| Airplane | $C_L$ (actual) | $C_L$ (predicted) | Actual landing speed (m.p.h.) | Predicted landing speed (m.p.h.) |
|---|---|---|---|---|
| Lockheed (Sirius) | 1.40 | 1.373 | 57.3<br>57.0<br>Av.(57.15) | 57.8 |
| Ford (4 AT-E) | 1.39 | 1.33 | 59.1<br>58.4<br>Av.(58.75) | 60.1 |
| Hamilton (H-45) | 1.282 | 1.260 | 54.8<br>61.2<br>59.5<br>Av.(58.5) | 59.0 |
| Stinson (Jr.) | 1.462 | 1.35 | 54.7<br>53.9<br>56.1<br>Av.(54.9) | 57.1 |
| SE-5A | 1.10 | 1.15 | 54.0 | 52.9 |
| JN-6H | 1.178 | 1.18 | 51.0 | 51.0 |
| VE-7 | 1.10 | 1.142 | 51.0 | 50.1 |
| DH-4b | 1.095 | 1.123 | 56.5 | 55.9 |
| Sperry Messenger | 1.22 | 1.163 | 44.0 | 45.1 |
| MB-3 | 1.142 | 1.182 | 57.0 | 56.0 |
| MB-2 | 1.098 | 1.195 | 58.0 | 55.6 |

The results of tests of two additional airplanes – the Fokker CO-4 and the S.P.A.D. VII – appear in Carroll's report; inability to obtain complete design characteristics prevented the prediction of their landing speeds. This is particularly unfortunate in the case of the S.P.A.D. since its landing lift coefficient is unusually small ($C_L = 0.98$). This value, however, would appear to be consistent with the use of a very thin airfoil of small camber in a biplane of low equivalent aspect ratio which has only a medium value of the landing angle.

On examination of the above table, it will be noted that there is a maximum variation predicted and experimental values of less than $2\frac{1}{2}$ miles per hour in landing speed and 10 per cent in lift coefficient. It would appear unreasonable to expect better agreement than this in view of the probable accuracy of the test data.

The experimental accuracy of the biplane results is stated as plus or minus 3 per cent. The principal sources of error in the results obtained by the writer are probably variation of the camera speed and of wind velocity during a test. Different calibrations of the camera showed noticeable differences in speed between different spools although no important variation was detected within any one. It will be remembered that the average over a short period, including the instant of contact, was used as the instantaneous wind velocity. If the wind were not steady, the instantaneous velocity might vary consider-

ably from this average. One other possibility for error is seen in the use of the no-load tire circumference in the calculations of the ground speed. It is necessarily small, however, because at the speed and attitude of landing, the load on the wheels can only be a very small fraction of the total weight of the airplane. In most cases the full load radial deflection of the tire corresponds to a reduction of circumference of not more than 4 or 5 per cent.

Errors in the predictions themselves may result from several causes, the principal one being the assumption of a fictitious wing area to compensate for the lift produced by the parasite parts. Other possible sources of error are failure to reproduce in the airplane the profile of the specified airfoil, inaccuracies in the model test data, and the uncertainty of assumed airfoil characteristics when no full scale data was available for the profile used. In consideration of the possible sources of inaccuracy in both the predicted and actual values, the method of prediction is considered highly satisfactory.

With the validation of the theoretical principles utilized for prediction, it becomes possible to enumerate and discuss the importance of the design characteristics which control airplane landing speeds. These variables are wing loading, airfoil profile characteristics, landing angle of attack, aspect ratio and wing height.

In reality there are only two entirely distinct variables,

wing loading and lift coefficient, as indicated by the funda-
mental equation

$$V = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\left(\frac{1}{C_L}\right)}.$$

No discussion of the first seems necessary. The lift coeffi-
cient attainable in landing is fixed by the characteristics of
the wing profile, the equivalent aspect ratio when close to the
ground, and the angle of attack in the three-point landing atti-
tude. The possibility of reducing landing speeds by the use of
slotted or other forms of high lift wings is clearly indicated,
and the necessity for consideration of the effective aspect ratio
in fixing the landing angle is likewise apparent. It will be
noticed that the theory indicates no effect of wing height upon
the landing speed which could not be obtained otherwise, that is,
by adjustment of the landing angle. The small influence of wing
height upon lift curve slope for the aspect ratios common to
existing monoplanes may be seen in the predictions for these
airplanes.

## C o n c l u s i o n.

The landing speed of an airplane can be predicted with sat-
isfactory accuracy by the use of accepted aerodynamic theory.

Note.- Generalized curves to facilitate the prediction of landing
        speeds are given in Appendix II.

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                                       port No. 167, 1925.

Appendix I

TABLE II. Collected Results (Monoplane)

| Airplane | Gross weight lb. | Wing area sq.ft. | Wing loading lb./sq.ft. | Ground speed m.p.h. | Wind speed m.p.h. | Actual landing speed m.p.h. | Predicted landing speed m.p.h. | $C_L$ (actual) | $C_L$ (predicted) |
|---|---|---|---|---|---|---|---|---|---|
| Lockheed (Sirius) | 3415 | 288 | 11.86 | 59.1 | -1.8 | 57.3 | | | |
| | | | | 59.0 | -2.0 | 57.0 | | | |
| | | | | | | Av.(57.15) | 57.8 | 1.40 | 1.373 |
| Ford (4AT-E) | 10085 | 8×18 | 12.32 | 59.1 | 0 | 59.1 | | | |
| | | | | 58.4 | 0 | 58.4 | | | |
| | | | | | | Av.(58.75) | 60.1 | 1.39 | 1.33 |
| Hamilton (H-45) | 4536 | 405 | 11.21 | 54.8 | 0 | 54.8 | | | |
| | | | | 57.7 | 3.5 | 61.2 | | | |
| | | | | 58.4 | 1.1 | 59.5 | | | |
| | | | | | | Av.(58.5) | 59.0 | 1.282 | 1.260 |
| Stinson (Jr.) | 2797 | 248 | 11.29 | 54.7 | 0 | 54.7 | | | |
| | | | | 52.4 | 1.5 | 53.9 | | | |
| | | | | 45.8 | 10.5 | 56.1 | | | |
| | | | | | | Av.(54.9) | 57.1 | 1.462 | 1.35 |

TABLE III. Collected Results (Biplane)

| Airplane | Gross weight lb. | Wing area sq.ft. | Wing loading lb./sq.ft. | Actual landing speed m.p.h. | Predicted landing speed m.p.h. | $C_L$ (actual) | $C_L$ (predicted) |
|---|---|---|---|---|---|---|---|
| SE-5A | 2080 | 254 | 8.19 | 54 | 52.9 | 1.10 | 1.15 |
| JN-6H | 2767 | 354 | 7.83 | 51 | 51.0 | 1.178 | 1.18 |
| VE-7 | 2152 | 295 | 7.29 | 51 | 50.1 | 1.10 | 1.142 |
| DH-4b | 4000 | 450 | 8.89 | 56.5 | 55.9 | 1.095 | 1.123 |
| Sperry Messenger | 965 | 160 | 6.03 | 44 | 45.1 | 1.22 | 1.163 |
| MB-3 | 2277 | 242 | 9.43 | 57 | 56.0 | 1.142 | 1.182 |
| MB-2 | 10520 | 1121 | 9.39 | 58 | 55.6 | 1.098 | 1.195 |

## Ford 4-A-T Prediction

### Specifications

High-wing cabin monoplane (trimotored).

| | |
|---|---|
| Wing profile | U.S.A. 27B at root<br>Göttingen 387 at tip* |
| Span | 74 ft. |
| Area | 818 sq.ft. |
| Height of wing | 9.5 ft. |
| Weight as flown | 10,085 lb. |
| Wing loading | 11.21 lb./sq.ft. |
| Ground angle | 18 deg. from 0 lift. |

### Ground Effect

$$\mu = 1$$

$$\frac{h_1 - h_1'}{b} = \frac{2 \times 9.5}{74} = 257$$

$$\sigma = .416$$

$$R_e = \frac{R}{1 - \sigma} = \frac{b^2}{S} \times \frac{1}{1 - \sigma}$$

$$= \frac{74^2}{818} \times \frac{1}{1 - .416} = 11.48$$

### Lift Curve Correction

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{11.48} - \frac{1}{6.85} \right) 57.3$$

$$= \alpha_1 - 1.075 C_L$$

*Göttingen 387 characteristics used.

| $\alpha_1$ (deg.) | $C_L$ | $-1.075C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| -6.8 | 0 | - 0 | -6.8 |
| -3 | .28 | - .301 | -3.3 |
| 0 | .50 | - .538 | -0.538 |
| 3 | .723 | - .776 | 2.224 |
| 6 | .951 | -1.021 | 4.979 |
| 9 | 1.153 | -1.236 | 7.764 |
| 12 | 1.305 | -1.403 | 10.597 |
| 15 | 1.330 | -1.430 | 13.570 |
| 18 | 1.318 | -1.417 | 16.583 |

### Hamilton H-45 Prediction

### Specifications

High-wing cabin monoplane

| | |
|---|---|
| Wing section | Clark Y modified* |
| Span | 54.25 ft. |
| Area | 405 sq.ft. |
| Height of wing | 7.75 ft. |
| Weight as flown | 4536 lb. |
| Wing loading | 11.21 lb./sq.ft. |
| Ground angle | 11 degrees |

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*Clark Y characteristics used.

## Ground Effect

$$\frac{h_{1-1}'}{b} = \frac{2 \times 7.75}{54.25} = .286 \qquad \mu = 1 \qquad \sigma = .388$$

$$R_e = \frac{R}{1-\sigma} = \frac{b^2}{S} \times \frac{1}{1-\sigma}$$

$$= \frac{54.25}{405} \times \frac{1}{1-.388} = 11.88$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{11.88} - \frac{1}{6.85}\right) 57.3 = \alpha_1 - 1.127 C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-1.127 C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| −5.2 | 0 | −0 | −5.2 |
| −3.0 | .162 | − .182 | −3.182 |
| 0 | .380 | − .428 | −0.428 |
| 3 | .600 | − .675 | 2.325 |
| 6 | .820 | − .924 | 5.076 |
| 9 | 1.033 | −1.065 | 7.935 |
| 12 | 1.230 | −1.386 | 10.614 |
| 15 | 1.366 | −1.538 | 13.462 |
| 18 | 1.284 | −1.448 | 16.552 |

Lockheed Sirius Prediction

## Specifications

Low-wing open monoplane

| | |
|---|---|
| Wing profile | tapered Clark Y* |
| Area | 288 sq.ft. |
| Span | 41.9 ft. |
| Height of wing | 4 ft. |
| Weight as flown | 3415 lb. |
| Wing loading | 11.86 lb./sq.ft. |
| Ground angle | 15 degrees |

## Ground Effect

$$\frac{h_{1-1}'}{b} = \frac{8}{41.9} = .191 \qquad \mu = 1 \qquad \sigma = .5$$

$$R_e = \frac{b^2}{S} \times \frac{1}{1-\sigma} = \frac{41.9}{288} \times \frac{1}{.5} = 12.18$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{12.18} - \frac{1}{6.85}\right) 57.3 = \alpha_1 - 1.162 C_L$$

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*Clark Y characteristics used.

| $\alpha_1$ (deg.) | $C_L$ | $-1.162C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| -5.2 | 0 | 0 | -5.2 |
| -3.0 | .162 | - .188 | -3.188 |
| 0 | .380 | - .442 | - .442 |
| 3 | .600 | - .699 | 2.301 |
| 6 | .820 | - .955 | 5.045 |
| 9 | 1.033 | -1.205 | 7.795 |
| 12 | 1.230 | -1.431 | 10.569 |
| 15 | 1.366 | -1.588 | 13.412 |
| 18 | 1.284 | -1.495 | 16.505 |

## Stinson Jr. Prediction

### Specifications

Cabin monoplane

| | |
|---|---|
| Wing profile | Clark Y |
| Span | 41.8 ft. |
| Area | 248 sq.ft. |
| Height of wing | 7.5 ft. |
| Weight as flown | 2797 lb. |
| Wing loading | 11.29 lb./sq.ft. |
| Ground angle | 13.1 degrees |

## Ground Effect

$$\frac{h_{1-1'}}{b} = \frac{15}{41.7} = .359$$

$$\mu = 1 \qquad \sigma = .320$$

$$R_e = \frac{R}{1 - \sigma} = \frac{b^2}{S} \times \frac{1}{1 - \sigma}$$

$$= \frac{41.7}{248} \times \frac{1}{1 - .320} = 10.3$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{10.3} - \frac{1}{6.85} \right) 57.3$$

$$= \alpha_1 - .875 \ C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-.875 C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| −5.2 | 0 | 0 | −5.2 |
| −3.0 | .162 | − .142 | −3.142 |
| 0 | .380 | − .323 | −0.323 |
| 3 | .600 | − .525 | 2.475 |
| 6 | .820 | − .718 | 5.282 |
| 9 | 1.033 | − .905 | 8.095 |
| 12 | 1.230 | −1.076 | 10.924 |
| 15 | 1.366 | −1.196 | 13.804 |
| 18 | 1.284 | −1.125 | 16.875 |

## Sperry Messenger Prediction

### Specifications

Open biplane (light one-seater)

| | |
|---|---|
| Wing profile | U.S.A.-5 |
| Area | 160 sq.ft. |
| Span (both wings) | 20 ft. |
| Gap | 2.83 ft. |
| Chord (both wings) | 4.0 " |
| Height of lower wing | 2.1 " |
| Weight as flown | 965 lb. |
| Wing loading | 6.03 lb./sq.ft. |
| Ground angle | 17.2 deg. |

### Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 3.83}{2 \times 26} = .1915$$

$$\mu = \frac{b_1}{b_2} = 1$$

$$\sigma = .498$$

Since it is of equal span and equal chord

$$\frac{1}{k^2} = \frac{1}{2}(1 + \sigma)$$

$$= \frac{1.498}{2} = .749$$

$$k^2 = 1.335$$

## Ground Effect

| Wings | h | $\dfrac{2\,h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|-----|-----|-----|------|
| 1 – 1' | 11.86 | .593 | 1 | .187 |
| 2 – 2' | 4.2 | .21 | 1 | .471 |
| 1 – 2' | 8.03 | .401 | 1 | .288 |

$$\overline{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\,\sigma_{1-2'}}{4}$$

$$= \frac{.187 + .471 + 2 \times .288}{4} = .3085$$

$$R_e = \frac{k^2\,b^2}{s} \times \frac{1}{1 - \overline{\sigma}\,k^2}$$

$$= \frac{1.335 \times 20}{160\,(1 - .3085 \times 1.335)} = 5.68$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{5.68} - \frac{1}{6.85}\right) 57.3 = \alpha_1 + .547 C_L$$

| $\alpha_1$(deg.) | $C_L$ | $.547C_L$ | $\alpha_2$(deg.) |
|---|---|---|---|
| -4.2 | 0 | 0 | -4.2 |
| 0 | .325 | .177 | 0.177 |
| 3 | .555 | .304 | 3.304 |
| 6 | .780 | .426 | 6.426 |
| 9 | 1.000 | .547 | 9.547 |
| 12 | 1.190 | .651 | 12.651 |
| 15 | 1.185 | .649 | 15.649 |
| 18 | 1.135 | .621 | 18.621 |

JN – 6H Prediction

## Specifications

Open biplane (primary training)

| | |
|---|---|
| Wing profile | Eiffel – 36* |
| Span, upper | 43.7 ft. |
| "      lower | 32.7  " |
| Gap | 5.17  " |
| Height of lower wing | 2.35  " |
| Wing area | 354    sq.ft. |
| Wing loading | 7.83 lb./sq.ft. |
| Ground angle of attack | 13.2  deg. |

---

*U.S.A.–5 characteristics used.

## Equivalent Monoplane

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 5.17}{43.7 + 32.7} = .1355$$

$$\mu = \frac{b_2}{b_1} = \frac{32.7}{43.7} = .748$$

$$\sigma = .52$$

$$\frac{1}{k^2} = \frac{1 - \sigma^2}{1 - 2 \sigma \mu + \mu^2} = \frac{1 - .52}{1 - 2 \times .52 \times .748 + .748}$$

$$= \frac{.73}{.781} = .935$$

$$k^2 = 1.07$$

## Ground Effect

| Wings | h | $\dfrac{2 h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|-------|-------|-------|-------|
| 1 – 1' | 15.04 | .344 | 1.0 | .330 |
| 2 – 2' | 4.70 | .1438 | .748 | .518 |
| 1 – 2' | 9.87 | .259 | 1.0 | .412 |

$$\overline{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4}$$

$$= \frac{.330 + .518 + 2 \times .412}{4} = .418$$

$$R_e = \frac{k^2 b^2}{s (1 - \overline{\sigma} k^2)}$$

$$= \frac{1.07 \times 43.7}{354 (1 - .418 \times 1.07)} = 10.47$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{10.47} - \frac{1}{6.85}\right) 57.3 = \alpha_1 - .922C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-.922C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| -2.2 | 0 | 0 | -2.2 |
| 0 | .163 | -.150 | -0.150 |
| 3 | .392 | -.362 | 2.638 |
| 6 | .622 | -.575 | 5.425 |
| 9 | .851 | -.785 | 8.215 |
| 12 | 1.063 | -.983 | 11.017 |
| 15 | 1.210 | -.118 | 13.882 |
| 18 | 1.01 | -.933 | 17.067 |

## SE – 5A Prediction

### Specifications

Open biplane (pursuit)

| | |
|---|---|
| Weight | 2080 lb. |
| Area | 254 sq.ft. |
| Wing loading | 8.19 lb./sq.ft. |
| Gap | 4.38 ft. |
| Span (both wings) | 2.66  " |
| Height of lower wing | 3.33  " |
| Landing angle | 14 deg. |
| Wing profile | R.A.F. 15 |

## Biplane Constant

$$\frac{2\ h}{b_1 + b_2} = \frac{4.38}{26.6} = .1648$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .538$$

$$\frac{1}{k^2} = \frac{1}{2}\ (1 + \sigma)$$

$$= \frac{1.538}{2} = .769$$

$$k^2 = 1.30$$

## Ground Effect

| Wings | h | $\frac{2\ h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|------|------|------|------|
| 1 – 1' | 17.42 | .580 | 1 | .192 |
| 2 – 2' | 6.67 | .250 | 1 | .422 |
| 1 – 2' | 11.04 | .415 | 1 | .279 |

$$\overline{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\ \sigma_{1-2'}}{4}$$

$$= \frac{.192 + .422 + 2 \times .279}{4} = .298$$

$$R_e = \frac{k^2\ b^2}{S\ (1 - \overline{\sigma}\ k^2)}$$

$$= \frac{130 \times 26.6}{254\ (1 - .298 \times 1.30)} = 5.93$$

### Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{5.93} - \frac{1}{6.85}\right) 57.3 = \alpha_1 + .373 C_L$$

| $\alpha_1$(deg.) | $C_L$ | $373 C_L$ | $\alpha_2$(deg.) |
|---|---|---|---|
| -2.2 | 0 | 0 | -2.2 |
| 0 | .163 | .061 | 0.061 |
| 3 | .392 | .146 | 3.146 |
| 6 | .622 | .232 | 6.232 |
| 9 | .851 | .318 | 9.318 |
| 12 | 1.063 | .397 | 12.397 |
| 15 | 1.210 | .451 | 15.451 |
| 18 | 1.010 | .377 | 18.377 |

### MB - 3 Prediction

### Specifications

Open biplane (pursuit)

| | |
|---|---|
| Wing section | R.A.F.-15 |
| Span, upper | 26 ft. |
| "      lower | 24.5 ft. |
| Gap | 4.5 " |
| Height of lower wing | 3.0 " |
| Wing area | 242 sq.ft. |
| Wing loading | 9.83 lb./sq.ft. |
| Ground angle of wings | 15 deg. |

## Biplane Constant

$$\frac{2\,h}{b_1 + b_2} = \frac{2 \times 4.5}{26 + 24.5} = 1.78$$

$$\mu = \frac{b_2}{b_1} = \frac{24.5}{26} = .942$$

$$\sigma = .508$$

$$\frac{1}{k^2} = \frac{1 - \sigma^2}{1 - 2\,\sigma\,\mu + \mu^2}$$

$$\frac{1}{k^2} = \frac{1 - .508}{1 - 2 \times .942 \times .508 + .942} = .800$$

$$k^2 = 1.25$$

## Ground Effect

| Wings | h | $\frac{2\,h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|---|---|---|---|---|
| 1 — 1' | 15 | .577 | 1.0 | .193 |
| 2 — 2' | 6 | .245 | .942 | .423 |
| 1 — 2' | 10.5 | .416 | 1.0 | .279 |

$$\overline{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\,\sigma_{1-2'}}{4}$$

$$= \frac{.193 + .423 + 2 \times .279}{4} = .293$$

$$Re = \frac{k^2\,b^2}{S(1 - \overline{\sigma}\,k^2)}$$

$$= \frac{1.25 \times 26}{232\,(1 - .298 \times 1.25)} = 5.51.$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{Re} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{5.51} - \frac{1}{6.85}\right) 57.3 = \alpha_1 + 6.47 C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $.647 C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| −2.2 | 0 | 0 | −2.2 |
| 0 | .163 | .106 | 0.106 |
| 3 | .392 | .254 | 3.254 |
| 6 | .622 | .403 | 6.403 |
| 9 | .851 | .551 | 9.551 |
| 12 | 1.063 | .680 | 12.680 |
| 15 | 1.210 | .783 | 15.783 |
| 18 | 1.010 | .654 | 18.654 |

## VE-7 Prediction

## Specifications

Open biplane (advanced training)

| | |
|---|---|
| Wing section | R.A.F.-15 |
| Area | 295 sq.ft. |
| Span (both wings) | 34.1 ft. |
| Gap | 4.67 ft. |
| Height of lower wing | 3.41 " |
| Weight as flown | 2152 lb. |
| Wing loading | 7.29 lb./sq.ft. |
| Ground angle | 12.7 deg. |

## Biplane Constant

$$\frac{2\,h}{b_1 + b_2} = \frac{2 \times 4.67}{2 \times 34.1} = .137$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .538$$

$$\frac{1}{k^2} = \frac{1}{2}\,(1 + \sigma) = .792$$

$$k^2 = 1.262$$

## Ground Effect

| Wings | h | $\dfrac{2\,h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|------|------|------|------|
| 1 − 1' | 16.14 | .474 | 1 | .243 |
| 2 − 2' | 6.8 | .1995 | 1 | .485 |
| 1 − 2' | 11.47 | .336 | 1 | .337 |

$$\overline{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\,\sigma_{1-2'}}{4}$$

$$= \frac{.243 + .485 + 2 \times .335}{4} = .3505$$

$$Re = \frac{k^2\,b^2}{S\,(1 - \overline{\sigma}\,k^2)}$$

$$= \frac{1.262 \times 34.1}{295\,(1 - .3505 \times 1.262)} = 8.96$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left( \frac{1}{8.96} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - .629 C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-.629 C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| −2.2 | 0 | 0 | −2.2 |
| 0 | .163 | −.102 | −0.102 |
| 3 | .392 | −.247 | 2.753 |
| 6 | .622 | −.391 | 5.609 |
| 9 | .851 | −.535 | 8.465 |
| 12 | 1.063 | −.680 | 11.320 |
| 15 | 1.210 | −.755 | 14.245 |
| 18 | 1.010 | −.635 | 17.365 |

## D—Hb Prediction

## Specifications

Open biplane (observation)

| | |
|---|---|
| Wing section | R.A.F.−15 |
| Area | 450 sq.ft. |
| Span (both wings) | 42.5 ft. |
| Gap | 5.5  " |
| Height of lower wing | 3,5  " |
| Weight as flown | 4000 lb. |
| Wing loading | 8.89 lb./sq.ft. |
| Ground angle | 12.3 deg. |

## Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 5.5}{2 \times 42.5} = .1295$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .590$$

$$\frac{1}{k^2} = \frac{1}{2} (1 + \sigma)$$

$$= .799$$

$$k^2 = 1.251$$

## Ground Effect

| Wings | h | $\frac{2 h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|------|-------|---|------|
| 1 - 1' | 18 | .424 | 1 | .274 |
| 2 - 2' | 7 | .165 | 1 | .537 |
| 1 - 2' | 12.5 | .294 | 1 | .375 |

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\,\sigma_{1-2'}}{4}$$

$$= \frac{.274 + .537 + 2 \times .375}{4} = .390$$

$$R_e = \frac{k^2\, b^2}{S\,(1 - \bar{\sigma}\, k^2)}$$

$$= \frac{1.251 \times 42.5}{450\,(1 - .390 \times 1.251)} = 9.76$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_1}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{9.76} - \frac{1}{6.85}\right) 57.3 = \alpha_1 - .793C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-.793C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| -2.2 | 0 | 0 | -2.2 |
| 0 | .163 | -.129 | -0.129 |
| 3 | .392 | -.311 | 2.689 |
| 6 | .622 | -.493 | 5.507 |
| 9 | .851 | -.675 | 8.325 |
| 12 | 1.063 | -.844 | 11.156 |
| 15 | 1.210 | -.960 | 14.040 |
| 18 | 1.010 | -.800 | 17.20 |

## Martin MB-2 Prediction

### Specifications

Open biplane (bimotored bomber)

| | |
|---|---|
| Wing section | Albatros* |
| Span (both wings) | 74.16 ft. |
| Gap | 8.5 " |
| Total wing area | 1121 sq.ft. |
| Weight as flown | 10520 lb. |
| Wing loading | 9.39 lb./sq.ft. |
| Height of lower wing | 4.5 ft. |
| Ground angle | 13 deg. |

*U.S.A.-5 characteristics used.

## Biplane Constant

$$\frac{2h}{b_1 + b_2} = \frac{2 \times 8.5}{2 \times 74.16} = .1147$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .620$$

$$\frac{1}{k^2} = \frac{1}{2}(1 + \sigma)$$

$$= \frac{1.628}{2} = .814$$

$$k^2 = 1.229$$

## Ground Effect

| Wings | h | $\frac{2h}{b_1 + b_2}$ | $\mu$ | $\sigma$ |
|-------|------|---------|---|-------|
| 1 – 1' | 26.0 | .350 | 1 | .328 |
| 2 – 2' | 9.0 | .1212 | 1 | .615 |
| 1 – 2' | 17.5 | .236 | 1 | .441 |

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\,\sigma_{1-2'}}{4}$$

$$= \frac{.328 + .615 + 2 \times .441}{4} = .456$$

$$R_e = \frac{k^2\, b^2}{S(1 - \bar{\sigma}\, k^2)}$$

$$= \frac{1.229 \times 74.16}{1121(1 - .456 \times 1.229)} = 10.75$$

## Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{R_e} - \frac{1}{R_{i1}}\right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi}\left(\frac{1}{10.75} - \frac{1}{6.05}\right) 57.3 = \alpha_1 - .969 C_L$$

| $\alpha_1$ (deg.) | $C_L$ | $-.969 C_L$ | $\alpha_2$ (deg.) |
|---|---|---|---|
| −4.2 | 0 | 0 | −4.2 |
| 0 | .325 | − .315 | −0.315 |
| 3 | .555 | − .538 | 2.462 |
| 6 | .780 | − .755 | 5.245 |
| 9 | 1.00 | − .969 | 8.031 |
| 12 | 1.190 | −1.152 | 10.848 |
| 15 | 1.185 | −1.150 | 13.850 |
| 18 | 1.135 | −1.100 | 16.900 |

# A p p e n d i x   II

The following curves (Fig. 21) have been prepared to facil-
itate the prediction of landing speeds.  They constitute a set
of generalized lift curves for effective aspect ratios of 4, 8,
and 12, and for airfoil sections whose maximum lift coefficients
are  1.0,  1.2, and 1.4.  To use these curves the effective
aspect ratio, the probable value of the maximum lift coefficient,
and the landing angle of attack (measured from the angle of zero
lift) must be known.  From these data and a lift coefficient
obtained by interpolation between the curves very satisfactory
results may be produced.

The appended table illustrates, by application of the curves
to the prediction of landing speeds for the airplanes treated
in this paper, the accuracy which may be expected.

TABLE IV. Collected Results from Use of Generalized Lift Curves

| Airplane | Wing loading lb./sq.ft. | $C_L$ at landing deg. | Actual landing speed m.p.h. | Predicted landing speed m.p.h. | $C_L$ (actual) | $C_L$ (predicted) | Re | $C_{L_{max}}$ |
|---|---|---|---|---|---|---|---|---|
| Lockheed (Sirius) | 11.86 | 20.2 | 57.15 | 51.8 | 1.40 | 1.37 | 12.18 | 1.39 |
| Ford-4AT | 12.32 | 18.0 | 58.75 | 60.3 | 1.39 | 1.32 | 11.48 | 1.33 |
| Hamilton (H-45) | 11.21 | 17.2 | 58.2 | 57.6 | 1.28 | 1.315 | 11.88 | 1.39 |
| Stinson (Jr.) | 11.29 | 18.3 | 54.9 | 57.1 | 1.46 | 1.35 | 10.3 | 1.39 |
| SE-5A | 8.19 | 16.2 | 54.0 | 53.5 | 1.10 | 1.12 | 5.93 | 1.21 |
| JN-6H | 7.83 | 15.4 | 51.0 | 51.5 | 1.178 | 1.16 | 10.47 | 1.21 |
| VE-7 | 7.29 | 14.9 | 51.0 | 50.5 | 1.10 | 1.12 | 8.96 | 1.21 |
| DH-4b | 8.89 | 14.5 | 56.5 | 55.9 | 1.095 | 1.12 | 9.76 | 1.21 |
| Sperry Messenger | 6.03 | 21.4 | 44.0 | 45.5 | 1.22 | 1.14 | 5.68 | 1.19 |
| MB-3 | 9.43 | 17.2 | 57.0 | 56.5 | 1.142 | 1.16 | 5.51 | 1.21 |
| MB-2 | 9.39 | 17.2 | 58.0 | 55.5 | 1.098 | 1.20 | 10.75 | 1.19 |

June, 1930.
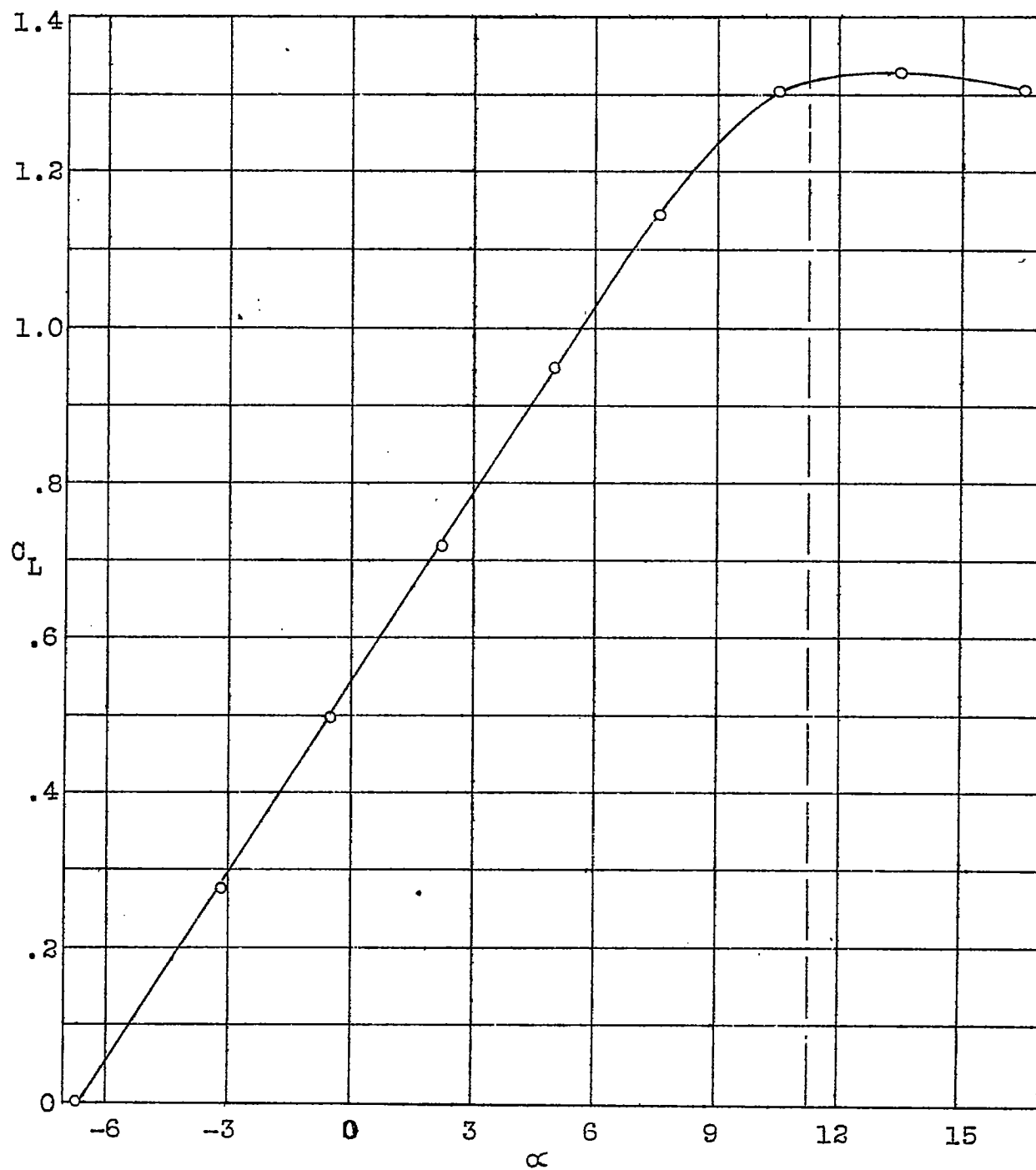
Fig. 1                              Ford 4-AT
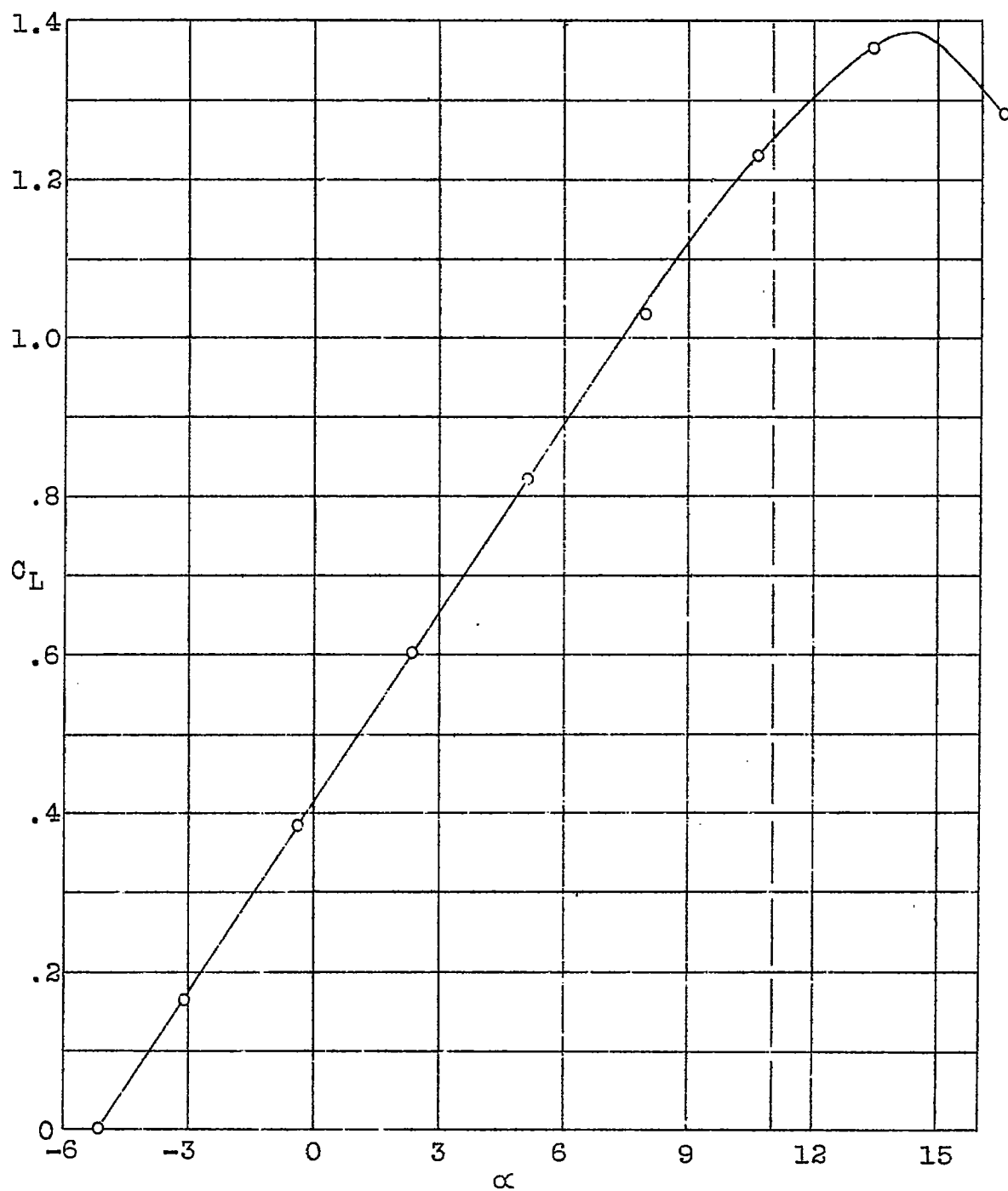
Fig. 2                    Hamilton H-45
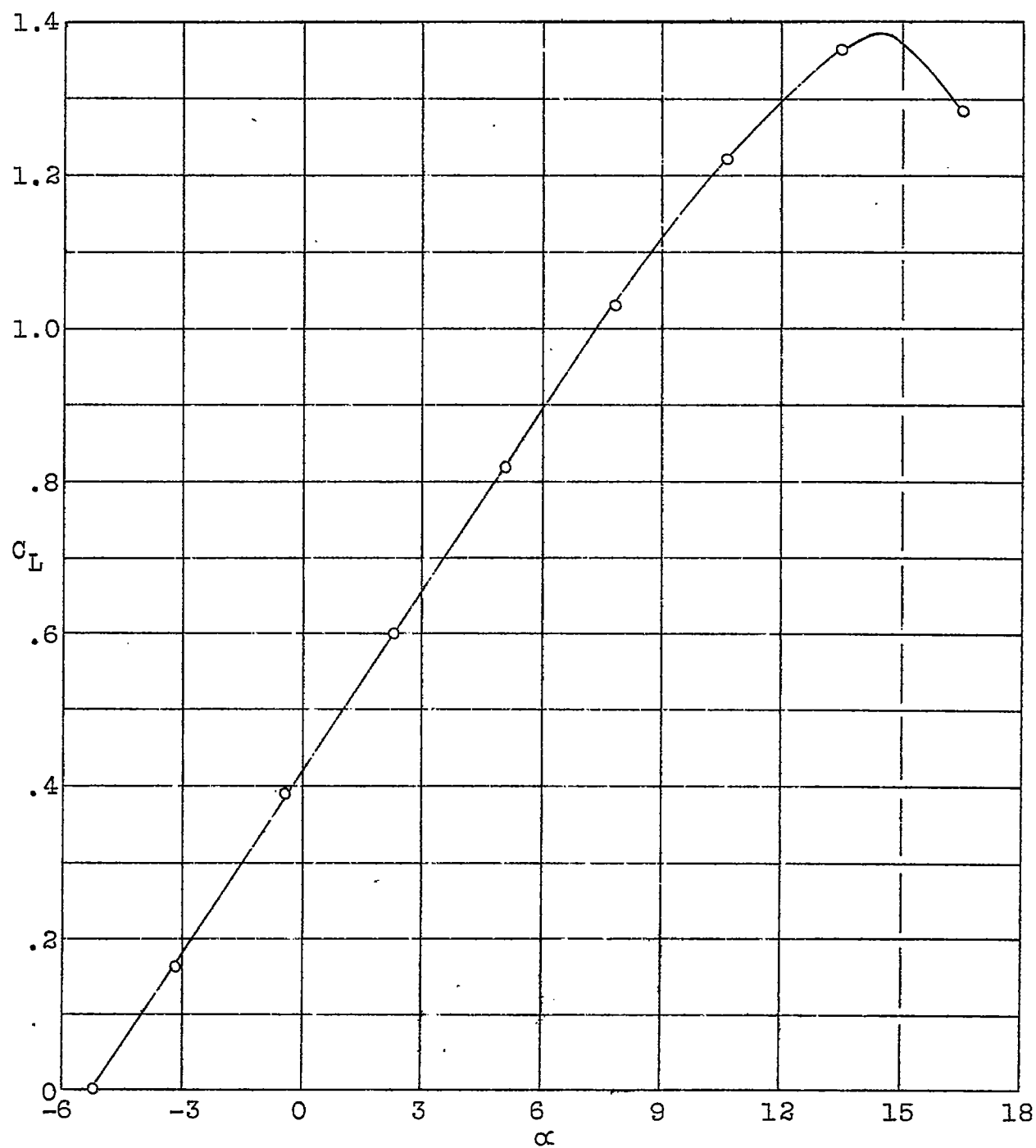
Fig. 3               Lockheed Sirius
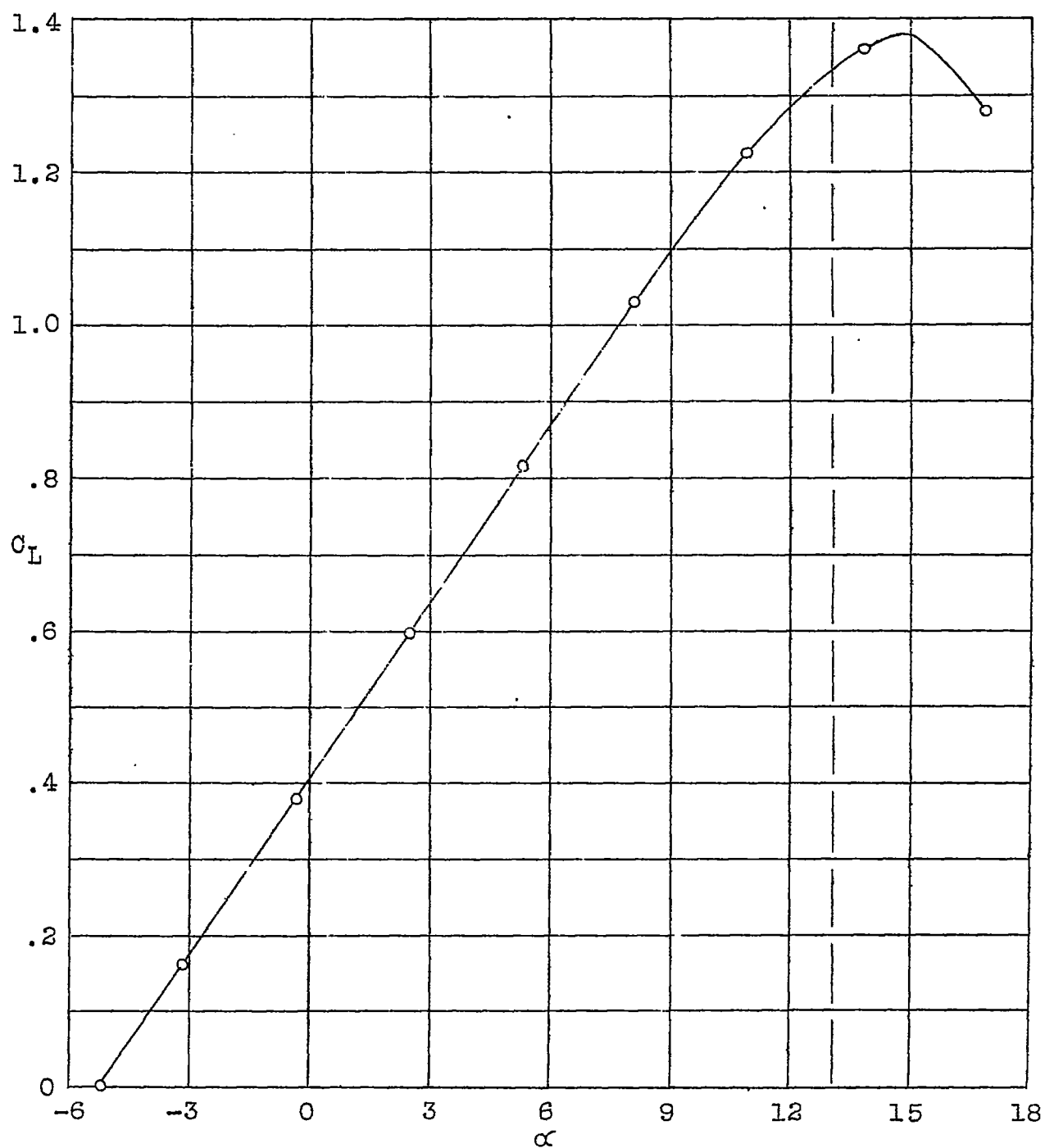
Fig. 4            Stinson Jr.
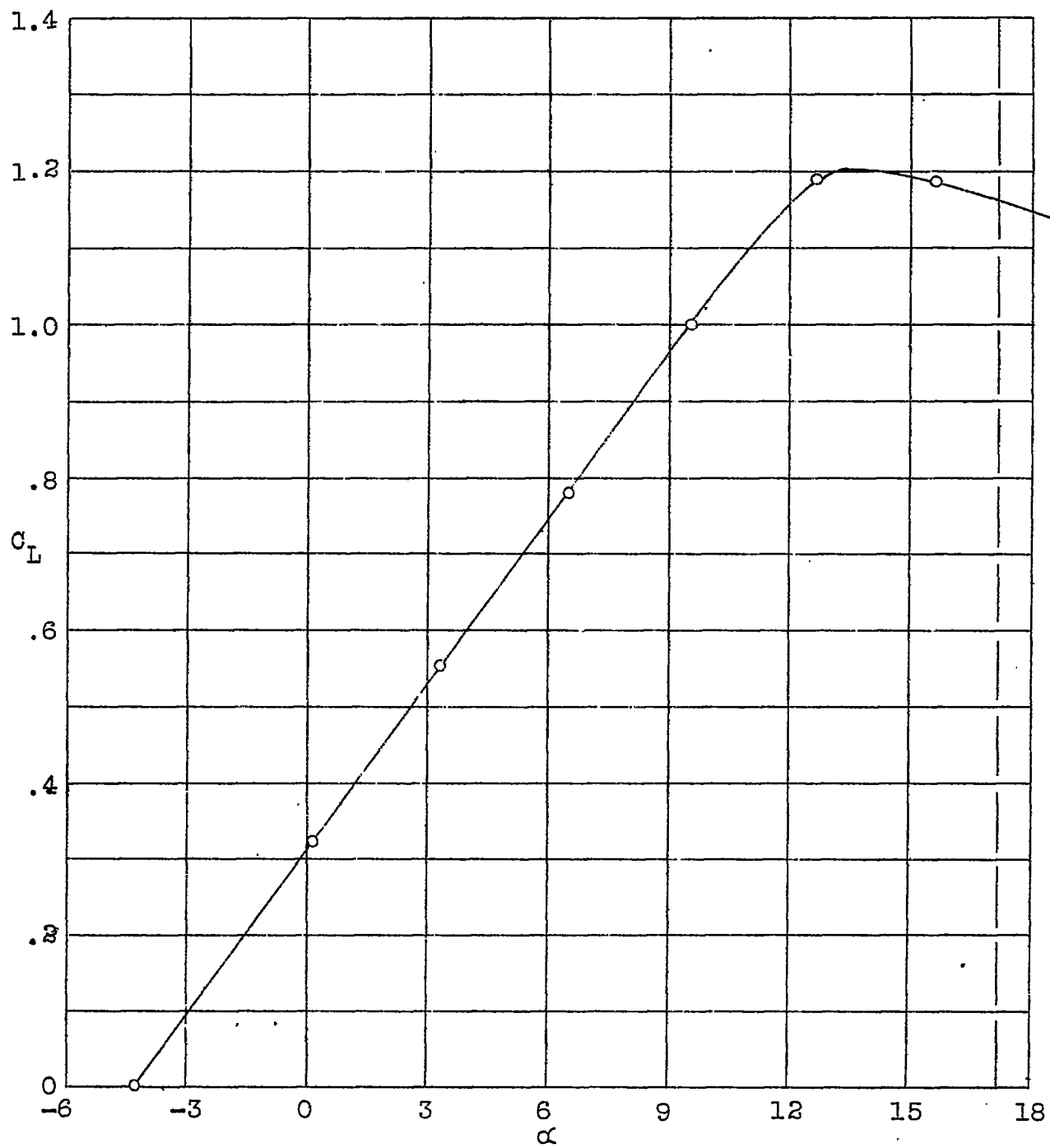
Fig. 5                    Sperry Messenger
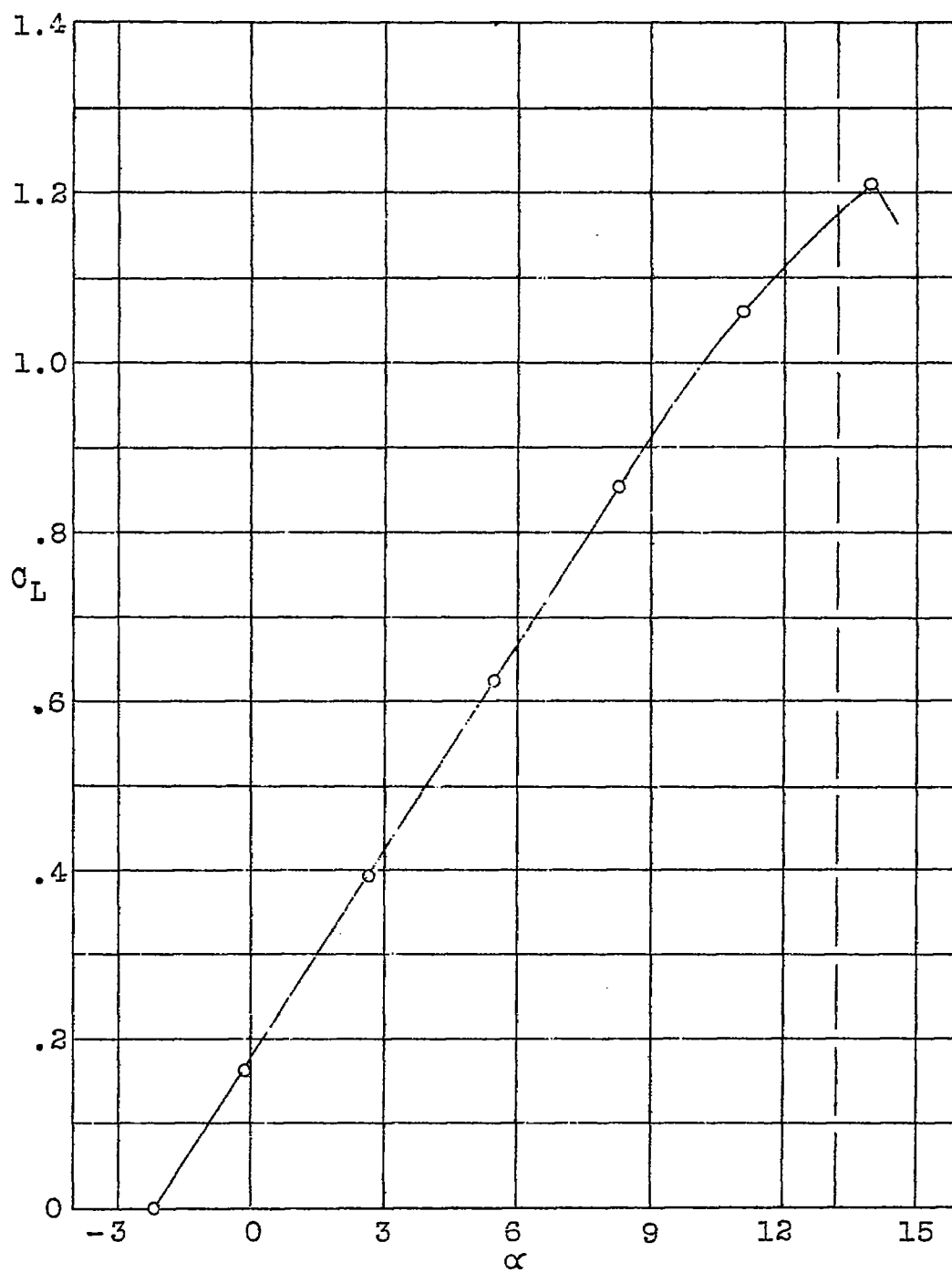
Fig. 6　　　　　　Curtiss JN 6H
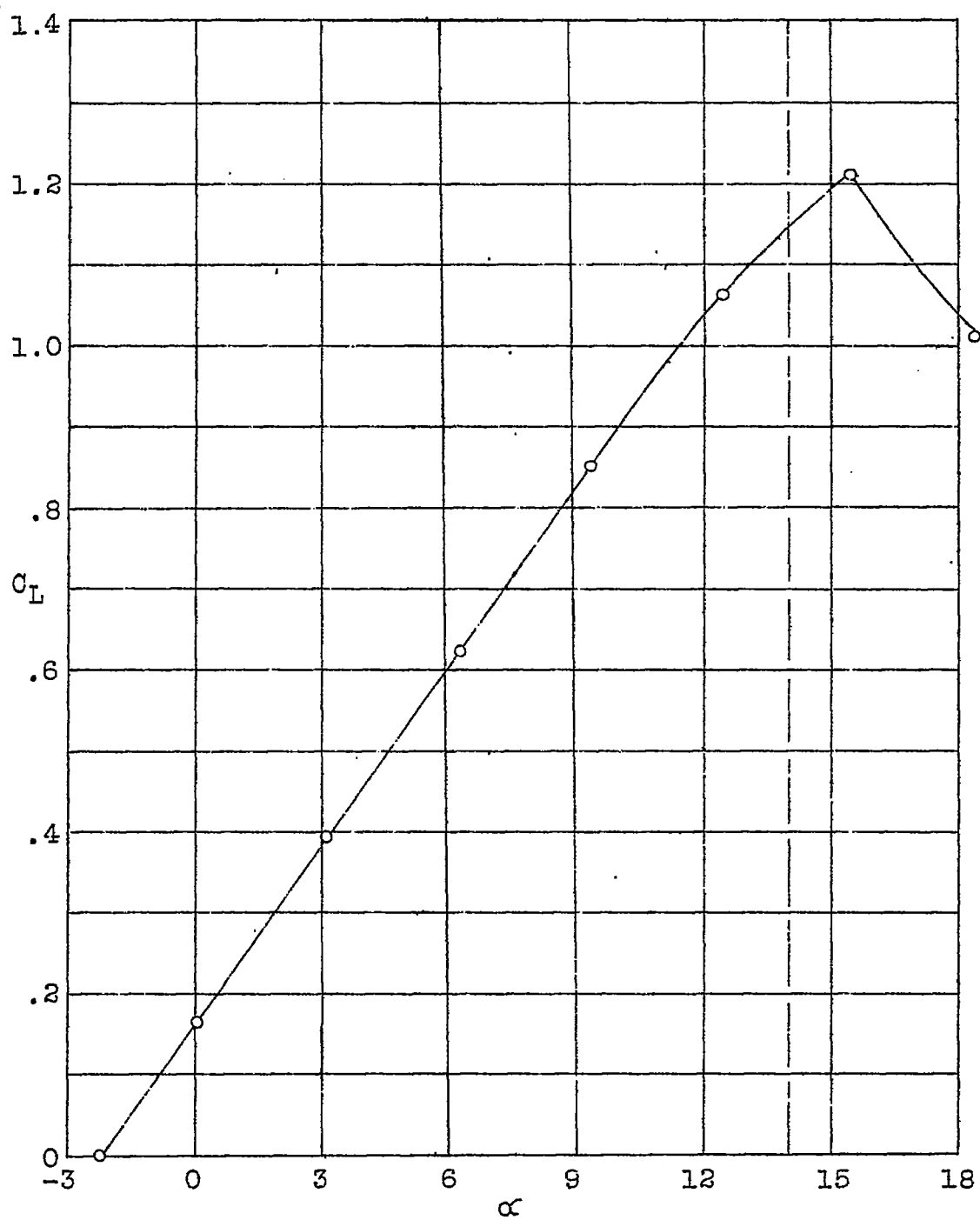
Fig. 7            SE-5A
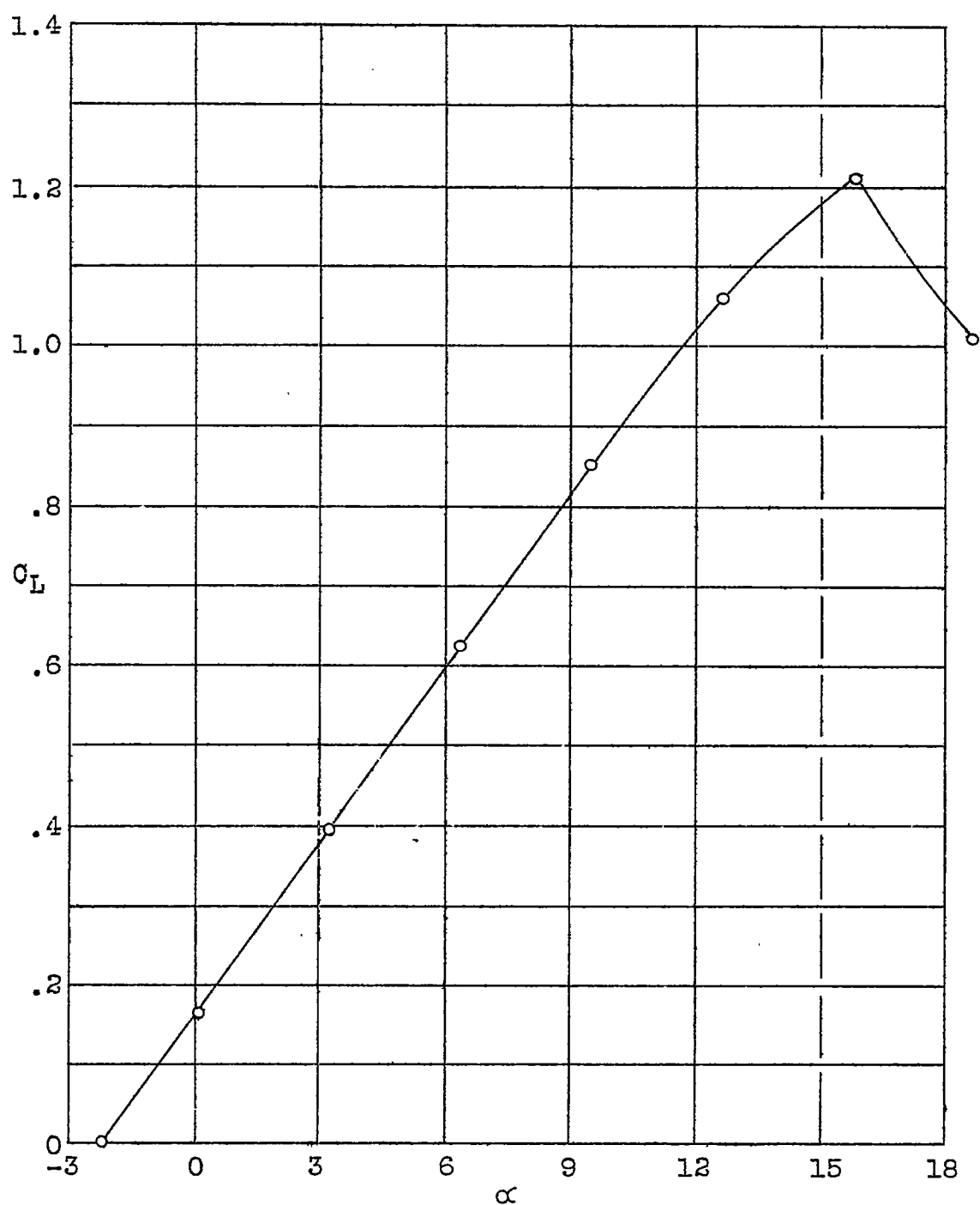
Fig. 8            Thomas Morse MB-3
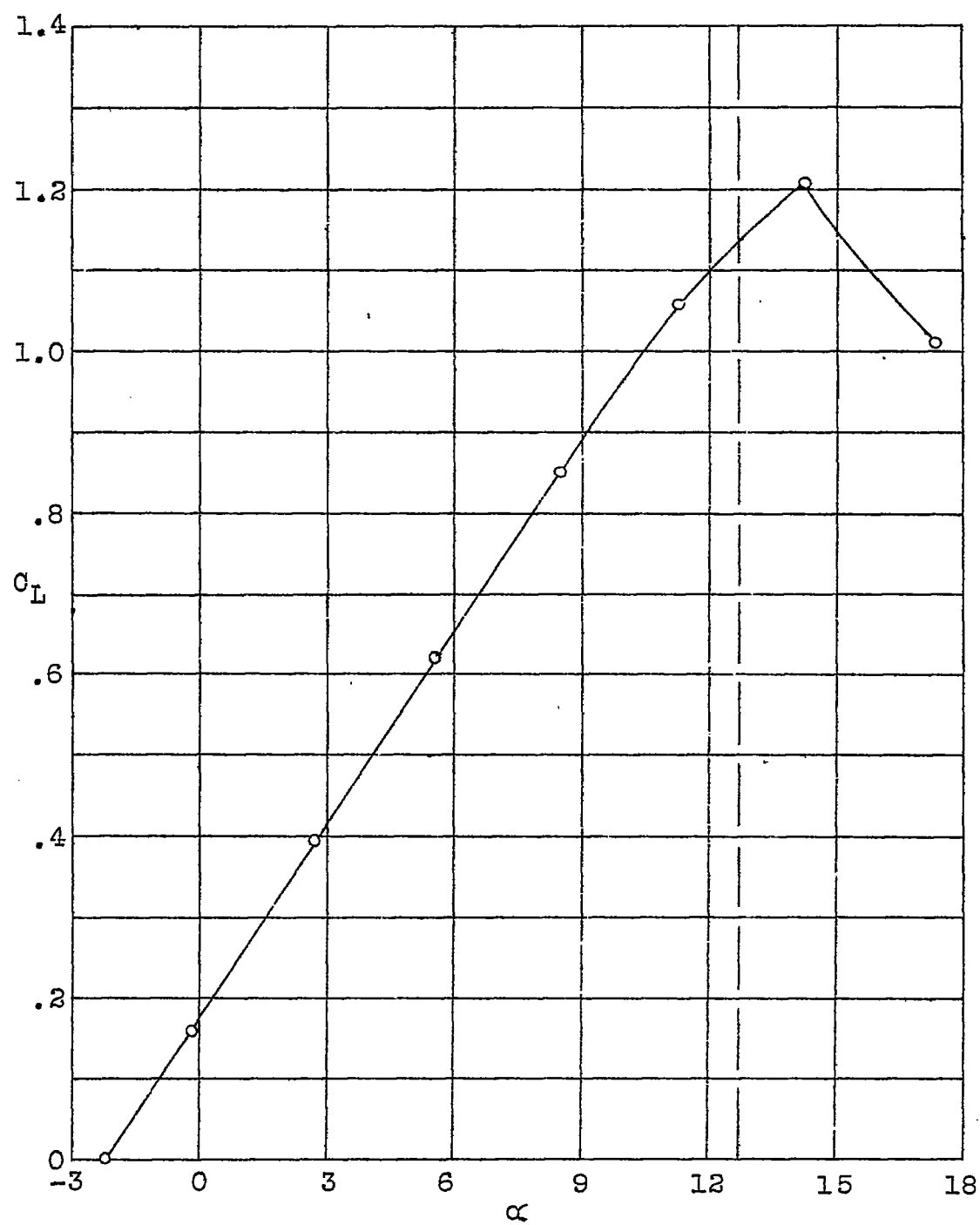
Fig. 9                              Vought VE-7
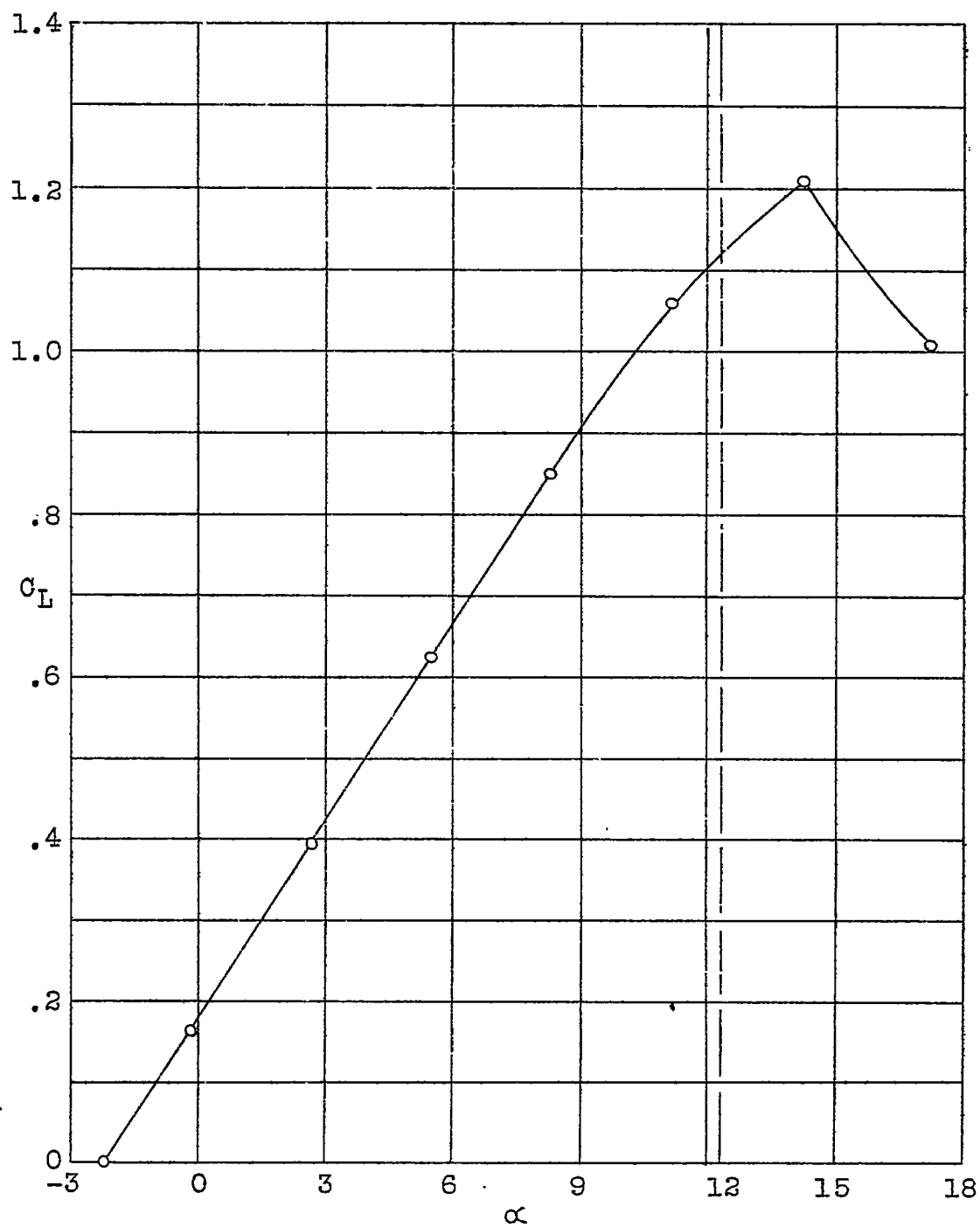
Fig. 10           DH-4B

Fig. 11　　　　　　　　Martin MB-II

Revolutions

Camera frame

Fig. 12

Ford 4-AT.

Fig. 12

Revolutions

0    4    8    12    16    20    24    28    32    36

Camera frame

Fig. 13

Hamilton H-45

Fig. 13

Fig. 14

Camera frame

Lockheed Sirius

Fig. 14

Fig. 15
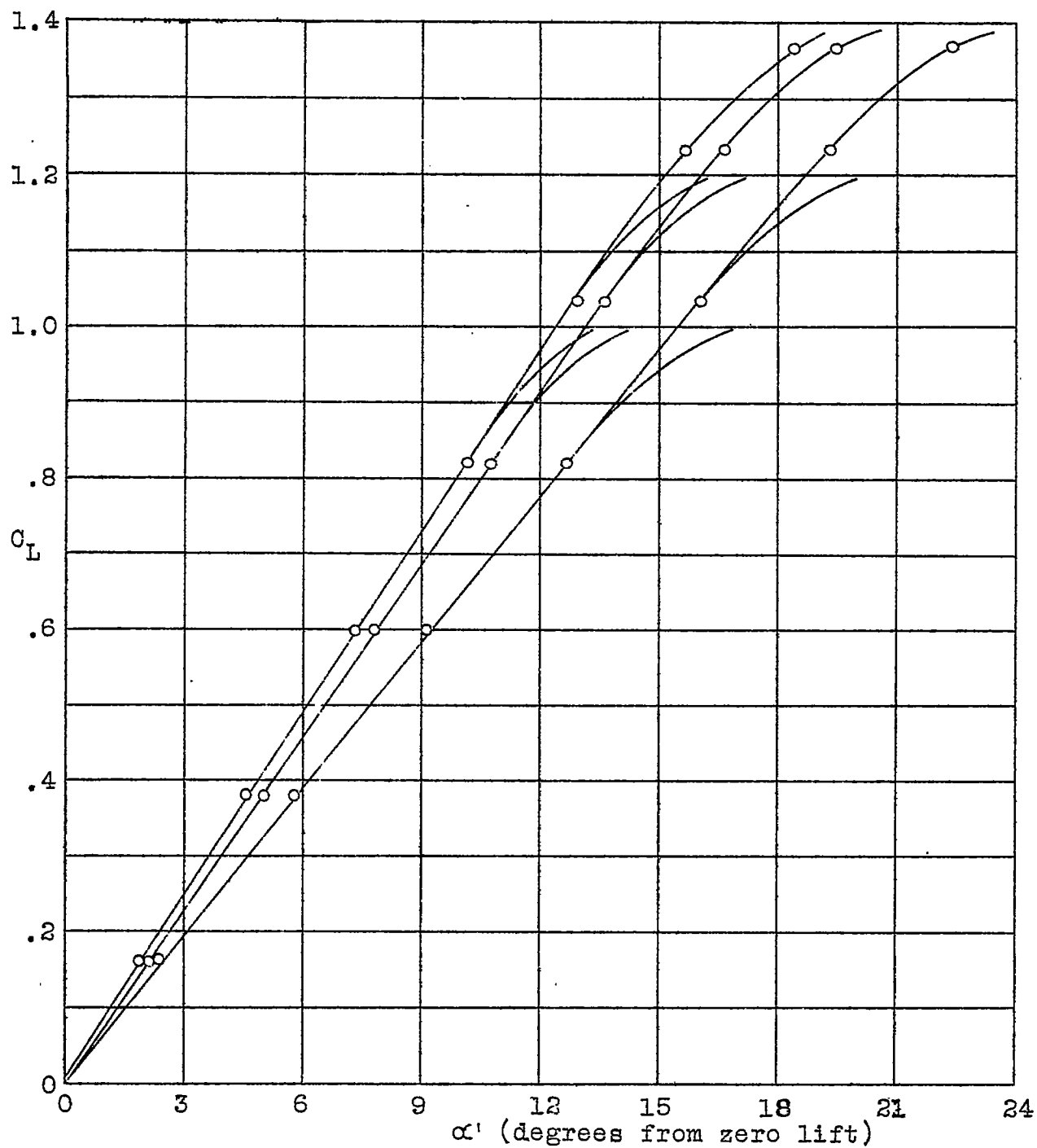
Camera frame

Stinson Jr.
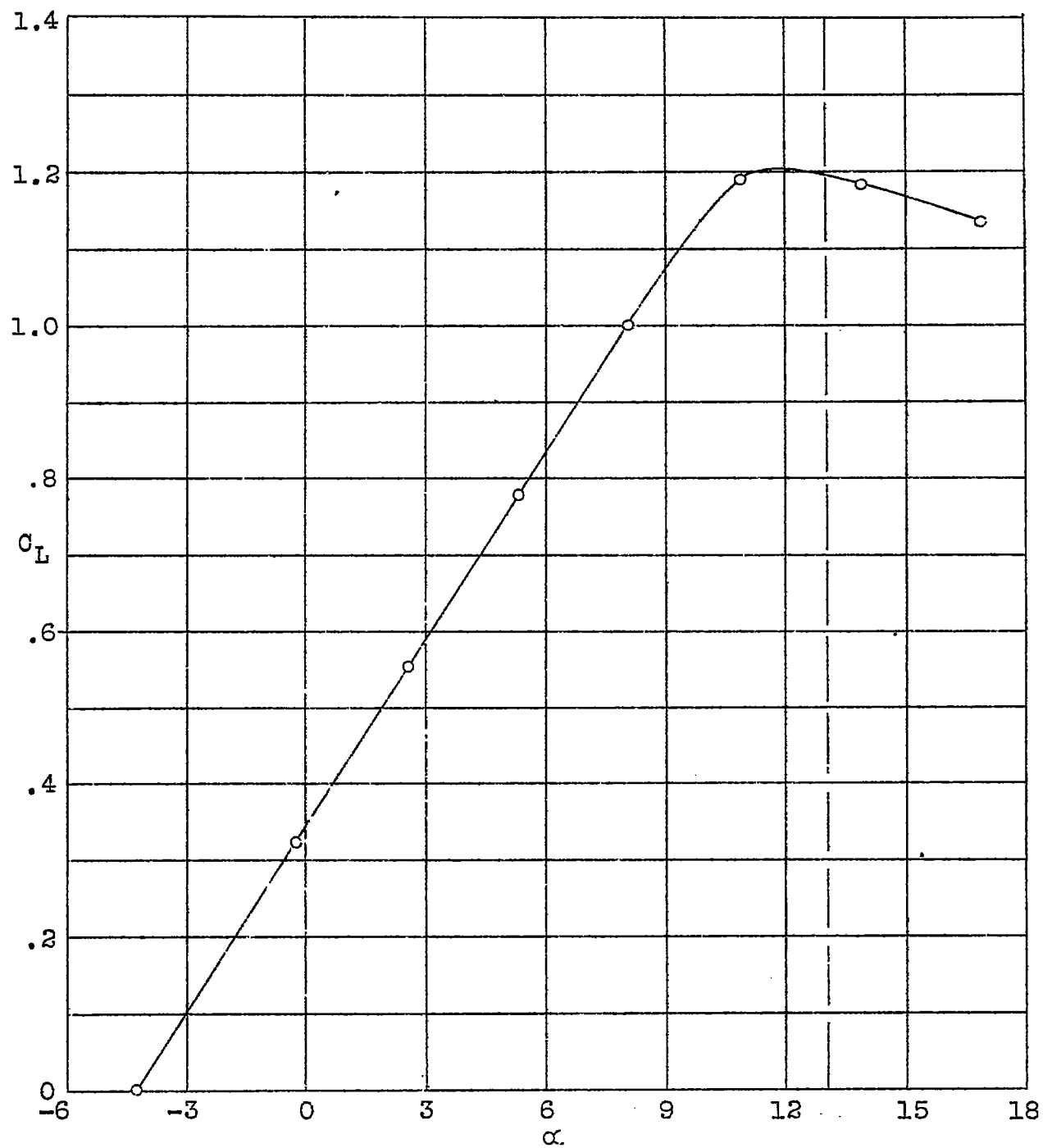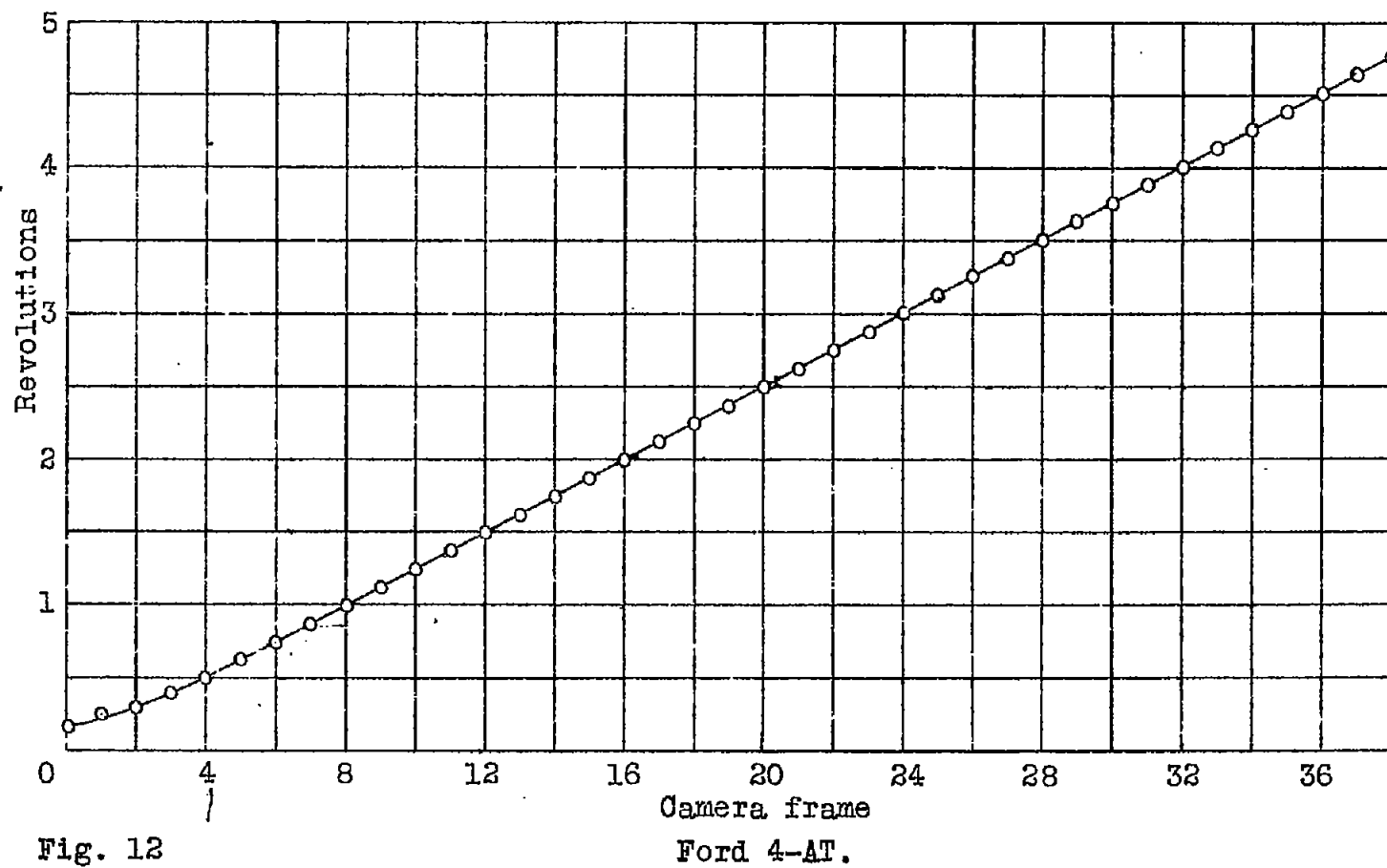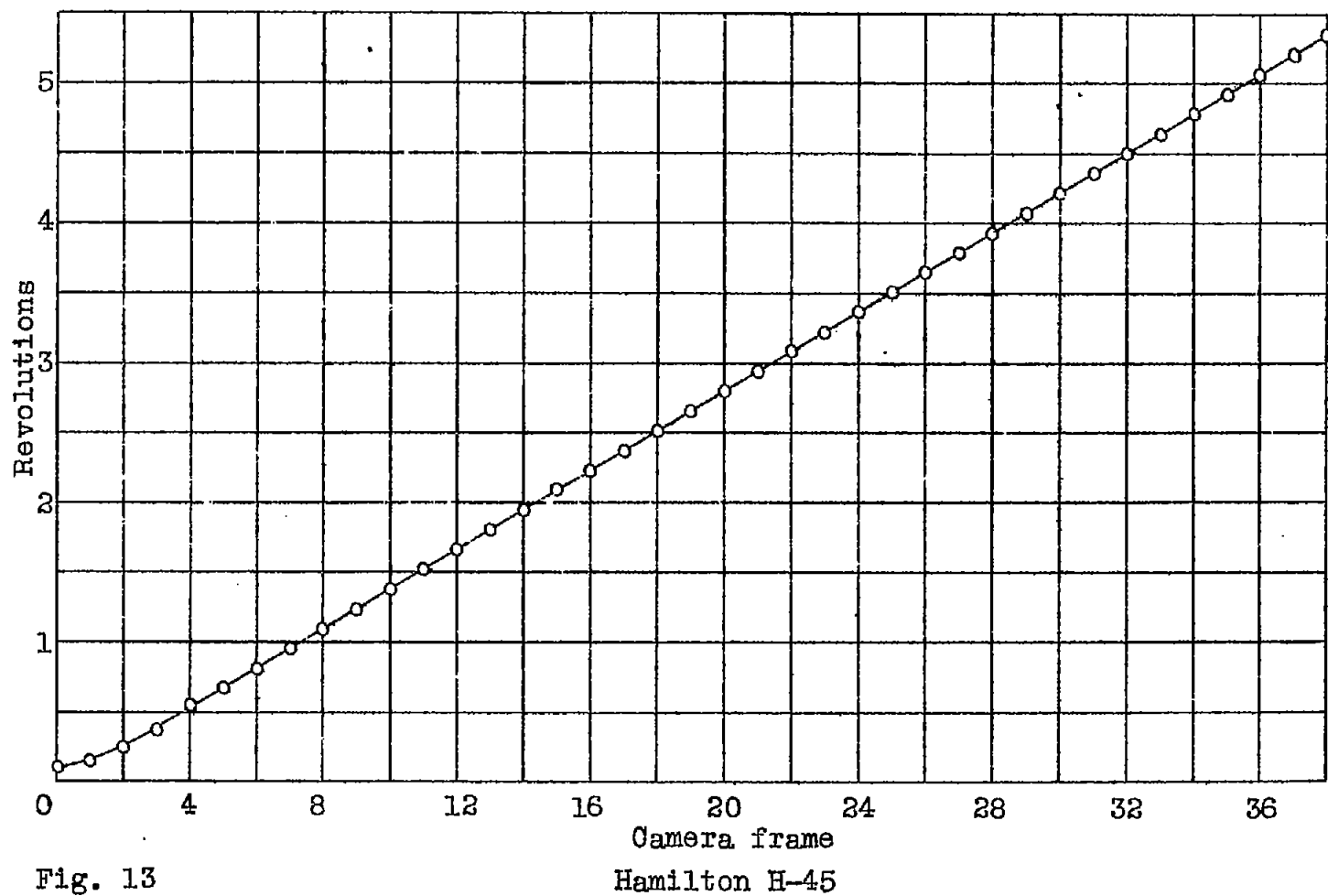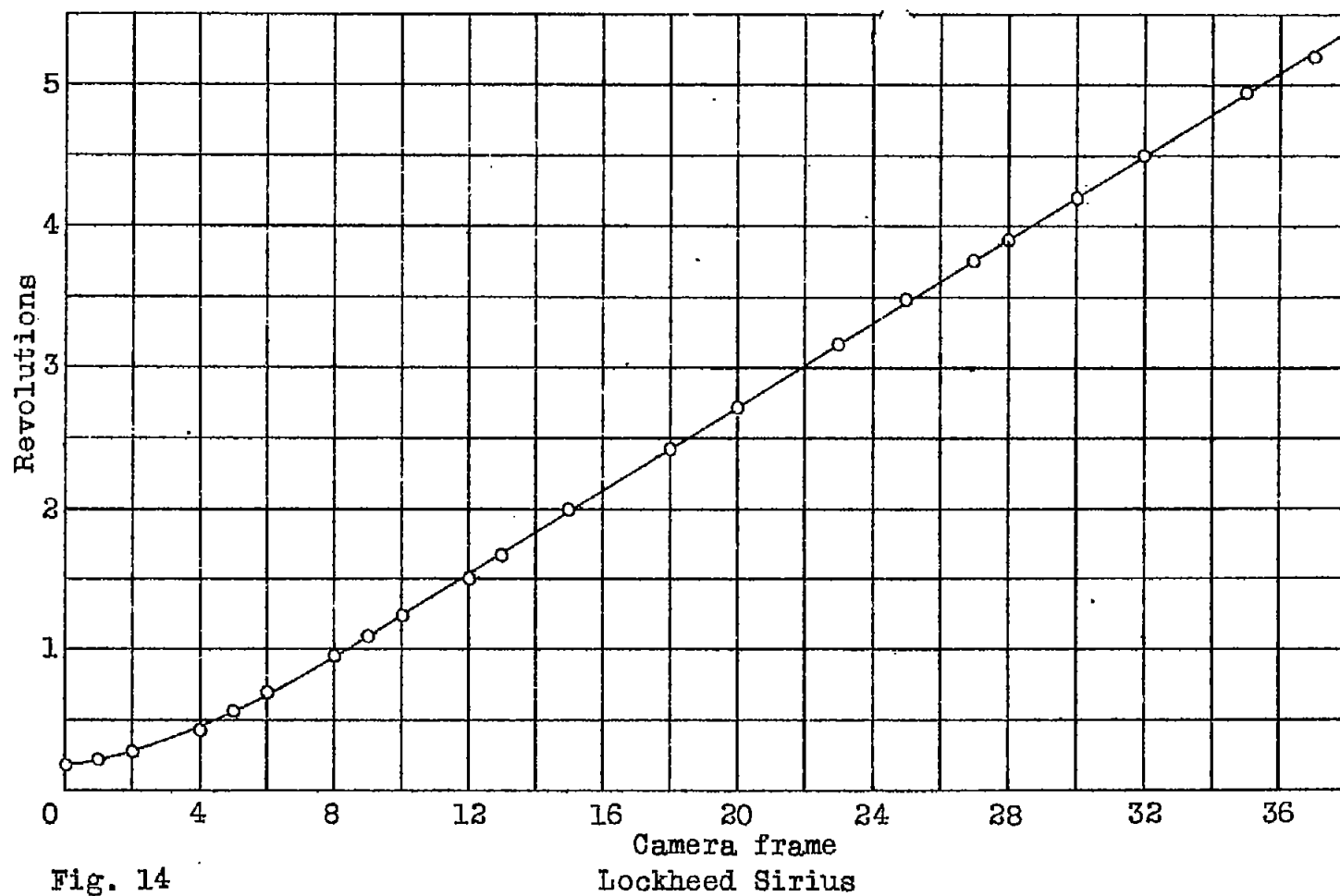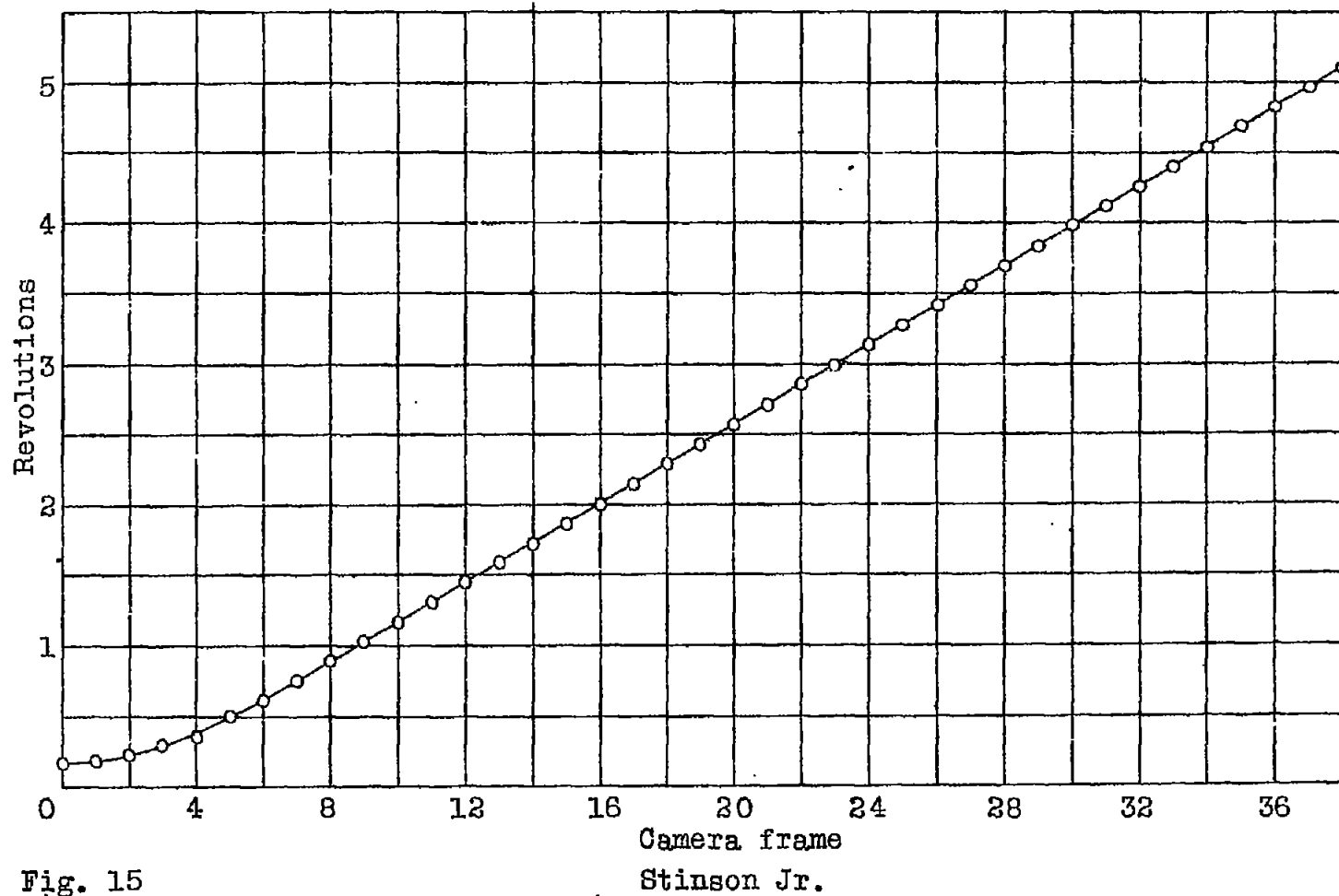
Fig. 15
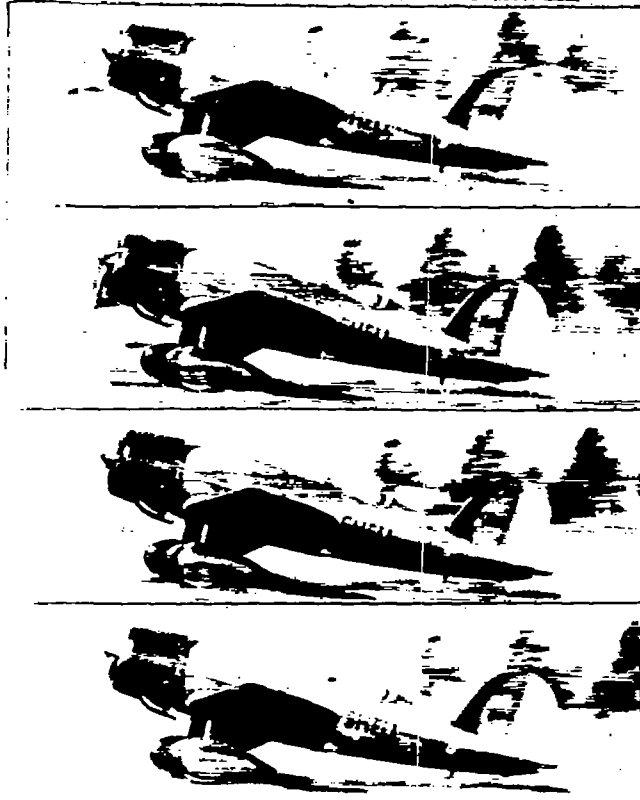
Fig. 16

Fig. 18

Fig. 17

Fig. 19

15626.A.S.
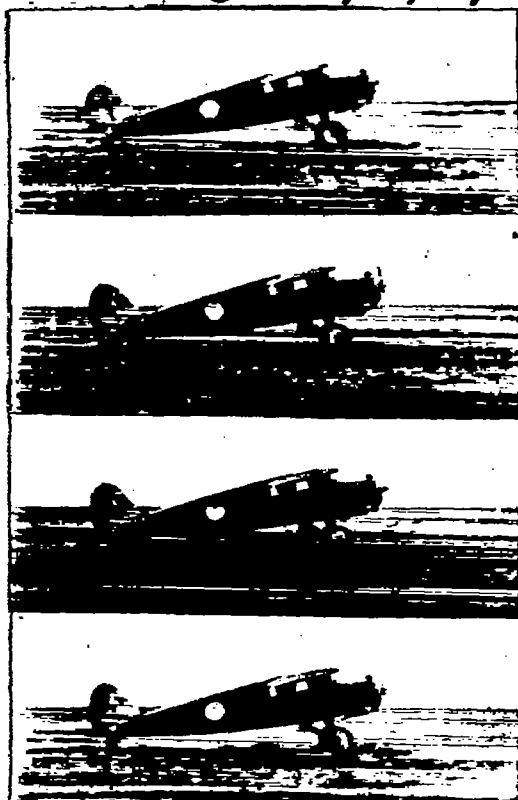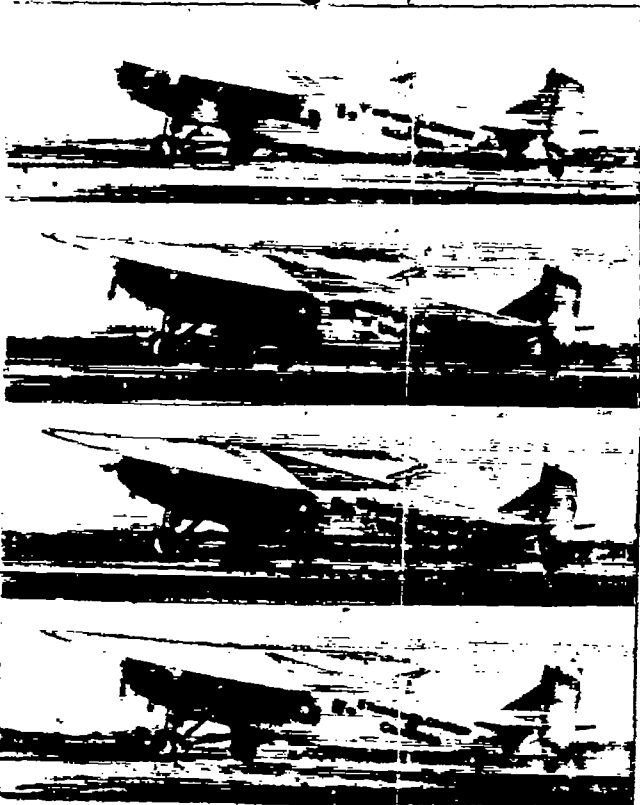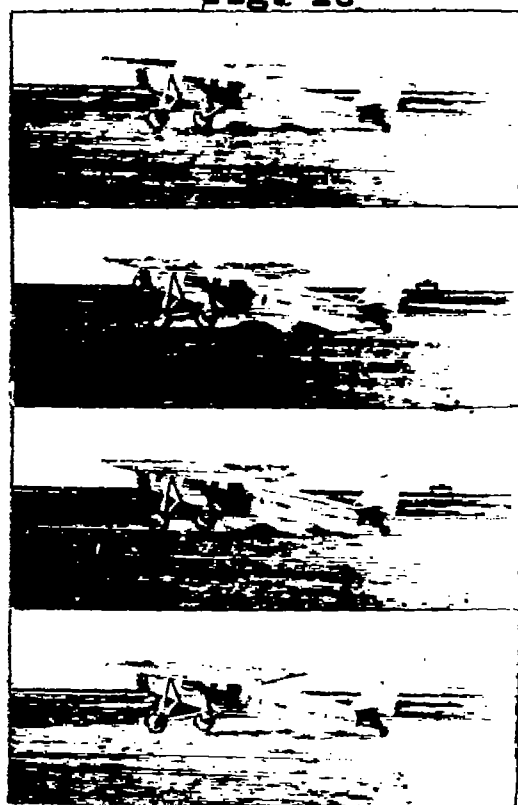
Fig. 20          Generalized lift curves.

Fig. 21

$$\frac{2h}{b_1 + b_2}$$

Fig. 21